DYNAMIC LEIDENFROST TEMPERATURE FOR UNDERHEATED DROPLETS

V. N. Mankevich and K. N. Agafonov

The effect of underheating of droplets on the critical conditions that arise when a surface is cooled by a stream of aerosol droplets is investigated. It is shown that in contrast to the static case, when underheating of droplets raises the critical temperature (relative to the saturation temperature) by a factor of 2-3, the corresponding increase in the dynamic situation is by a factor of tens.

The efficiency of cooling heated surfaces with streams of aerosol droplets is determined to a considerable measure by the mechanism of the interaction between the droplets and the surface. The familiar heat-exchange crisis: when the surface temperature exceeds some critical value, which depends on the physical and regime parameters of the cooling process, the heat-transfer coefficient decreases rapidly, other conditions being equal. This is due to the change from a regime of spreading and complete evaporation of droplets on the surface to regimes of recoils from the surface. A phenomenological description of such a phenomenon and the indicated heat-exchange crisis can be found, e.g., in [1, 2].

A number of theoretical studies [3, 4, 5] on the problem, however, were partial and the critical conditions corresponding to the onset of a change in the heat-exchange regimes were not determined. The problem was examined more fully in [6] where for the first time the conditions for the onset of the heat-exchange crisis were formulated and the dynamic Leidenfrost temperature determining that crisis was found; this was done with some assumptions that did not distort the fundamenetal picture of the phenomenon but at the same time permitted a substantial simplification of the very complex situation arising during the thermal and dynamic interaction of droplets with the over heated surface. Let us recall the conditions for a droplet to recoil from a rough surface with protuberances with a height [6]

$$\Delta \leqslant L_h \eta_{\min}(\text{We}, \varepsilon).$$

(1)

The equality sign in (1) corresponds to the onset of the crisis, i.e., direct contact of a droplet with the overheated wall. In this case, the other known parameters being equal, the equation stemming from (1) determines the dynamic Leidenfrost temperature T_L, which is equal to the wall temperature T_w corresponding to the onset of the crisis. From (1) it follows that

$$T_L \simeq T_s + \frac{4}{3} \frac{\sigma L R_0}{v''\lambda''} \varepsilon_L^4, \quad \frac{\Delta}{R_0} = \varepsilon_L \eta_{\min}(\text{We}, \varepsilon).$$

(2)

Equation (2) determines the Leidenfrost temperature for droplets in equilibrium with the vapor-gas medium, i.e., heated to T_s. At the same time, in most cases the surface is cooled by a stream of aerosol, whose temperature corresponds to the ambient temperature, i.e., $T_0 \leq T_s$. For such droplets the Leidenfrost temperature T_L can substantially exceed T_s from (2). A theory for underheated droplets carefully lowered to the surface, is given in [7]. As a result, underheating was shown to be the principal factor responsible for the considerable rise in T_L. Clearly, such an effect can also be expected for under heated droplets interacting dynamically with a heated wall. Our aim is to determine this effect.

Formulation of the Problem. The principal assumptions are similar to those in [6]. A disk-shaped droplet moving toward the wall was considered. Under conditions when the change in the volume of the droplet as the result of evaporation can be ignored, this makes it possible to describe the shape of the droplet with a single variable, the radius R of the liquid

---

disk, instead of a large number of variables. The second determinable variable is the layer thickness h.

We determined the two independent variables by using the law of conservation of energy in differential form

$$dE + dE^* + dU = f dh,$$ (3)

as well as Newton's second law of motion for the droplet as a whole

$$\rho' V (d^2 z_c / dt^2) = f.$$ (4)

The system of equations for determining R and h, which was obtained from (3) and (4), has the form [6]

$$\rho' V \left(\frac{d^2 h}{dt^2} - 2 \frac{R_0^3}{R^3} \frac{d^2 R}{dt^2} + 6 \frac{R_0^3}{R^4} \left(\frac{dR}{dt} \right)^2 \right) = f,$$ (5)

$$\rho' V \left[\left(1 + \frac{8}{3} \frac{R_0^6}{R^6} \right) \frac{d^2 R}{dt^2} - 8 \frac{R_0^6}{R^7} \left(\frac{dR}{dt} \right)^2 \right] + 8\pi\sigma \left(R - \frac{R_0^3}{R^2} \right) = 4 \frac{R_0^3}{R^3} f.$$

The force of the excess pressure is determined from the solution of the known quasistationary Reynolds problem [8] and in the general case depends on two terms [9]

$$f = \frac{3\pi \nu'' R^4}{2h^3} \left(j - \rho'' \frac{dh}{dt} \right).$$ (6)

The first term in (6) determines the force due to the evaporation of a droplet and the second is due to squeezing of the vapor-gas mixture when evaporation does not occur at all. It is easily seen that at $T_0 = T_S$ we usually have $\rho'' dh/dt \ll j$, i.e., the force is determined by the first term. At the same time, in the case of underheating $(T_0 \lesssim T_S)$ evaporation can be disregarded and the force acting on the droplet is written as

$$f = - \frac{3}{2} \pi \frac{\nu'' R^4}{h^3} \rho'' \left| \frac{dh}{dt} \right|.$$ (7)

As in [6], as $t \to -\infty + 0$ we have an asymptotic equality $R = R_0$, $dR/dt = 0$, $h = \infty - 0$, $dh/dt = dz_c/dt = -v_0$. In specific calculations it is desirable to change the origin and to consider the following relations as the conditions:

$$R \simeq R_0, \quad h \simeq h_0, \quad dR/dt \simeq 0, \quad dh/dt \simeq -v_0, \quad t = 0.$$ (8)

We introduce the dimension less variables

$$x = R/R_0, \quad \eta = h/L_h, \quad \tau = t/L_t,$$ (9)

which, respectively, character the degree of spreading of the droplet, the thickness of the vapor layer, and the duration of the collision process. Representations for the scales L_h and L_t are obtained in the standard way upon reducing system (5) and the initial conditions (8) to a dimensionless form. Taking Eq. (7) for f into account and going over to the variables (9), we determine

$$L_h = \left(\frac{9}{8} \frac{\nu'' \rho''}{\rho'} \right)^{1/2} \left(\frac{\rho' R_0^3}{6\sigma} \right)^{1/4}, \quad L_t = \left(\frac{\rho' R_0^3}{6\sigma} \right)^{1/2}.$$ (10)

We note that the scale L_h differs substantially from that in [6], where it had the form

$$L_h = \left(\frac{3}{4} \frac{\nu'' \lambda'' \Delta T}{\sigma L} \right)^{1/4} R_0^{3/4}.$$

From (5) and (8) we obtain the system of dimension less equations

$$2\left(1+\frac{8}{3x^6}\right)\frac{d^2x}{d\tau^2}-\frac{16}{x^7}\left(\frac{dx}{d\tau}\right)^2+x-\frac{1}{x^2}=-\frac{x}{\eta^3}\frac{d\eta}{d\tau},$$

$$\varepsilon\frac{d^2\eta}{d\tau^2}-\frac{2}{x^3}\frac{d^2x}{d\tau^2}+\frac{6}{x^4}\left(\frac{dx}{d\tau}\right)^2=-\frac{x^4}{\eta^3}\frac{d\eta}{d\tau}$$

(11)

and the initial conditions

$$x\simeq 1,\ \eta=\eta_0>1/\varepsilon,\ dx/d\tau=0,\ \frac{d\eta}{d\tau}\simeq-\sqrt{We}/\varepsilon,\ \tau=0.$$

(12)

Here we have introduced two dimension less parameters $We=\dfrac{\rho' R_0 v_0^2}{6\sigma}$ and , $\varepsilon=L_h/R_0$. System (11)

differs from the analogous sytem in [6] only by the right side. The modulus sign in (7) has been inserted because the right side changes sign when the droplet moves away from the wall.

Clearly, as the droplet approaches the wall the right side of (11) is equal to $\dfrac{x^4}{\eta^3}\dfrac{d\eta}{d\tau}$.

Numerical Solution and Discussion. System (11) was solved numerically for different values of parameters We and ε, which were chosen on the basis of the following considerations. As follows from [10], at $We\geq 5$ the droplet is subdivided and, therefore, the calculations were carried out for We values of 0 to 3. The value of ε is of the order of 10^{-2}-10^{-1}.

One of the most important assumptions of the model is that it is quasistationary, which allows Eq. (7) to be applied. Clearly, if this condition is to be satisfied we must have $h^2/a''/\ll h/(dh/dt)$; taking the reference for h and t into account, we obtain

$$\frac{L_h^2}{a''L_t}\eta\frac{d\eta}{d\tau}\ll 1.$$

(13)

For water $L_h\sim 10^{-4}$-10^{-5} m, $L_t\sim 10^{-2}$ sec, $a''\sim 10^{-5}$ m^2/sec, and condition (13) is equivalent to $\eta\ d\eta/d\tau\ll 10$, which obviously is satisfied when

$$\eta\frac{d\eta}{d\tau}\sim O(1).$$

(14)

We note that in [3, 4] we considered the dynamic problem for heated droplets on condition that h = const. In this case the thickness of the vapor layer, generally speaking, remains arbitrary where upon a condition of the type of (13) cannot be formulated since the natural scale L_h is missing. Nor can the conditions for determining the Leidenfrost temperature be formulated in this case. Such studies, of course, cannot be considered as coming up to the present status of the problem.

Since heating formally has no effect on the droplet dynamics, satisfaction of conditions (14) were checked by the results of numerical integration for system (11).

As shown by calculations, as the droplet approaches the surface condition (14) is not satisfied in the initial part of the fall, i.e., the thermal calculation cannot be carried out with the quasistationary model. On the other hand, as can be easily seen heating during this part of the fall is small and can be disregarded..

As the drop further approaches the surface the dynamic regime of the drop abruptly changes and from a certain time condition (14) begins to be satisfied. We note that this time can be tracked very clearly so tht the passage to condition (14) is virtually instantaneous.

Starting from this distance droplet heating can be calculated by the method of [7]. To distinguish the principal anomalies of droplet heating it is sufficient to consider the two limiting cases, when the characteristic time of temperature equalization inside the droplet R_0/a' is much shorter than the characteristic heating time (in this case the droplet temperature at any time can be assumed to be uniform) and the reverse situation, when the droplet
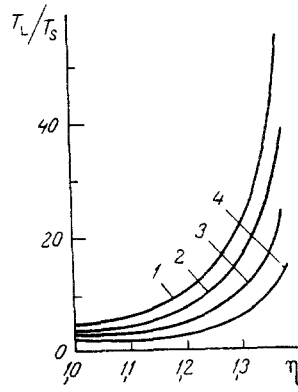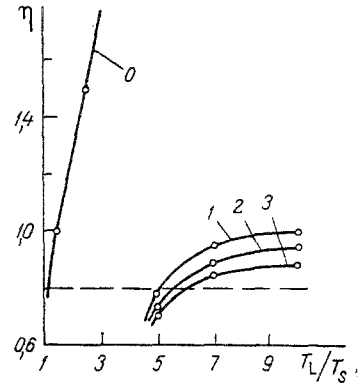
Fig. 1.   Fig. 2.

Fig. 1. Relative Leidenfrost temperature versus η for
unevenly heated droplets at different underheatings:
1) $T_0/T_S = 0.2$; 2) 0.4; 3) 0.6; 4) 0.8.

Fig. 2. Determination T_L/T_S for a given distance from
the surface (height of the roughness): 0) drops gently
settling onto the surface; 1) We = 1; 2) 2; 3) 3.

heating can be considered as heating of an unbounded half-space. In explicit form the cri-
terion for the first case is similar to condition (14), $R_0^2/a' \ll h/(dh/dt)$ or

$$\frac{R_0^2}{L_h a'} \ll 1. \tag{15}$$

It is easily seen that condition (15) is certainly not satisfied for water at $R \simeq 10^{-3}$ m. Thus
we must consider only the second situation of heating of a droplet as a half-space with the
initial and boundary conditions [7]

$$-\frac{\partial T}{\partial z} = \frac{\lambda''}{\lambda'} \frac{T_w - T}{h} \bigg|_{z=0}, \quad T \rightarrow T_0|_{z \rightarrow \infty}, \quad T = T_0|_{t=0}.$$

When h varies with time at a rather slow rate, i.e., when condition (14) is satisfied, the
formulated problem reduces to that considered in [11]. The temperature T^* at the lower sur-
face of the droplet in this case is

$$T^* = T_0 + (T_w - T_0)\left[1 - \exp\left(\frac{\lambda''^2}{\lambda'^2} \frac{a't}{h^2}\right) \text{erfc}\left(\frac{\lambda''}{\lambda'} \frac{\sqrt{a't}}{h}\right)\right]. \tag{16}$$

The Leidenfrost temperature will be defined as the surface temperature at which the droplet,
reaching the height Δ of the roughness, manages to heat up to T_S.

Taking scales L_h and L_S into account, we introduce the dimensionless variables η and
τ into (16), which we then rewrite as

$$\frac{T_L}{T_s} = \frac{1 - (T_0/T_s)\,\varphi}{1 - \varphi}, \quad \varphi = \exp\left(10^{-2}\frac{\tau}{h^2(\tau)}\right)\text{erfc}\left(10^{-1}\frac{\sqrt{\tau}}{h(\tau)}\right). \tag{17}$$

The values 10^{-2} and 10^{-1} in (17) are obtained upon substitution of the numerical values of
L_h and L_t for water.

The calculations from Eq. (17), generally speaking, must be carried out at h = const.
Since this formula is used in the quasistationry case, when the height h varies slowly with
time, the arguments $10^{-2}\tau/h^2(\tau)$ and $10^{-1}\sqrt{\tau}/h(\tau)$ are calculated as the integrated averages
in the period from the onset of the quasistationry regime to the next height (which in this
case was taken to be the height of the roughness).

The results of calculations from (17) for different values of underheating T_0/T_S and
We = 1 are shown in Fig. 1. We see that the curves are similar to those for underheated

droplets, gently settling onto an over heated surface [7]. Only the slope of the curves changes; this indicates that the temperature T_L for dynamic drops is higher than for drops gently settling onto the surface. While for the latter situation T_L/T_s does not exceed 2-2.5, in the dynamic case this quantity can increase tens of times (in the same range of the arguments). The last conclusion seems entirely natural.

The change in the situation is illustrated in Fig. 2. Taking a certain roughness (horizontal dashed line) as an example, we obtain the respective values of T_L/T_s, i.e., the Leidenfrost temperature, at the intersection of this line with different curves.

As for determining the temperature to which the droplet is heated during collision with the overheated wall, we note that an attempt was made in [5], where the balance method was used to determine $\Delta T = T^* - T_0$. This is permissible when the droplet heats up rapidly. As shown by our estimates, this situation does not obtain in the case of dynamic interaction of the droplet with the overheated wall and hence the procedure of [5] is not an adequate physically possible situation. Clearly, in this case we obtain overstimted results. We also note that from the practical standpoint the value of $\Delta T = T^* - T_0$ is less important than the critical temperature T_L; accordingly, it is more urgent to develop the results reported here to refine T_L. This generalization can be made in order to allow for the molecular slip at the boundary of the vapor layer, the dispersion interaction forces, and the dependence of the equilibrium temperature at the evaporation surface on the pressure near it [12].

NOTATION

a' and a'' thermal diffusivity of the liquid and the vapor; ε, a dimensionless parameter; ε_L, a parameter pertaining to T_L; E, kinetic energy of the droplet as a whole; E^*, kinetic energy of internal flows, corresponding to the deformation of the droplet; f, total force of the excess pressure acting on the droplet; h, thickness of the vapor layer; j, mass of liquid evaporated per unit area of the lower surface of the droplet per unit time; L, heat of the phase transition; L_h and L_t, characteristic linear and time scales; R and R_0, radii of the vapor layer and the spherical droplet; t, time; T_L, T_w, T_s, T^*, and T_0 Leidenfrost temperature, the temperature of the heated surface, the saturation temperature, the temperature of the lower surface of the droplet, and the initial temperature of the droplet, respectively; U, potential energy of the surface tension; v_0, initial velocity; V, volume of the droplet; We, modified Weber number; z and z_c, normal coordinate and the coordinate of the center of gravity; Δ, height of the roughness, η_{min}, minimum dimensionless thickness of the vapor layer; λ' and λ'', thermal conductivities of the liquid and the vapor; v' and v'', kinematic viscosities of the liquid and the vapor; ρ' and ρ'', densities of the liquid and the vapor; σ, a parameter introduced into (17); and is the coefficient of surface tension.

LITERATURE CITED

1. O. A. Kremnev and A. A. Satanovskii, Air/Water-Evaporation Cooling of Equipment [in Russian], Moscow (1967).
2. M. A. Styrikovich, V. S. Polonskii, and T. V. Tsiklauri, Heat and Mass Transfer and Fluid Mechanics in Two-Phase Flows of Atomic Power Plants [in Russian], Moscow (1982).
3. D. A. Kazenin and A. A. Makeev, Teor. Osn. Khim. Tekhnol., 20, No. 5, 653 (1986).
4. D. A. Kazenin and A. A. Makeev, Teplofiz. Vys. Temp., 25, No. 2, 317 (1987).
5. A. A. Gukhman, A. Z. Volynets, A. V. Rozhdestvenskii, and S. N. Rodionov, Inzh.-Fiz. Zh. 47, No. 5, 731 (1984).
6. Yu. A. Buevich, V. N. Mankevich, and M. I. Polotskii, Teplofiz. Vys. Temp., 24, No. 4, 743 (1986).
7. Yu. A. Buevich and V. N. Mankevich, Inzh.-Fiz. Zh. 44, No. 6, 949 (1983).
8. L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Pergamon, Oxford (1959).
9. Yu. A. Buevich and V. N. Mankevich. Mathematical Models, Analytical and Numerical Methods in Transport Theory [in Russian], Minsk (1982), pp. 99-107.
10. L. H. J. Wachters and N. A. J. Westerling, Chem. Eng. Sci., 21, No. 11, 1047 (1966).
11. T. Carslow and D. Jaeger, Thermal Conductivity of Solids [Russian translation], Moscow (1964).
12. Yu. A. Buevich and V. N. Mankevich, Teplofiz. Vys. Temp., 20, No. 6, 1136 (1982).